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Data-Transport Performance Analysis of Fasnet

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This paper presents a queueing model to assess the performance of Fasnet, a recently invented local-area network. Fasnet is intended for high-speed lines capable of carrying a wide mix of traffic. We confine our attention to data traffic only. An approximate formula for the expected delay of a packet is obtained; the approximate formula comparea favorably to simulations of Fasnet.

I. INTRODUCTION

Fasnet is an implicit token-passing local-area network.[1] It is intended for high-speed lines capable of carrying a wide mix of traffic (data, voice, video, and facsimile). In this paper, we present a queueing model to assess the performance of Fasnet with data traffic only. An approximation for the expected delay of a packet is obtained; the approximate solution compares favorably with measurements taken from a simulation of Fasnet. Our numerical results yield a mean delay that is less than 1 ms for a 1-kb packet when the line speed is 100 Mb/s and the occupancy of the line is 0.9.

Section II consists of a brief description of Fasnet. Section III describes our model and its approximate solution and Section IV presents comparisons with simulations. The effects of bursty traffic are given in Section V and our conclusions are stated in Section VI.

II. A BRIEF DESCRIPTION OF FASNET

We will now give a description of Fasnet that will enable the reader to appreciate the model in Section III. A complete description is given in Ref. 1.

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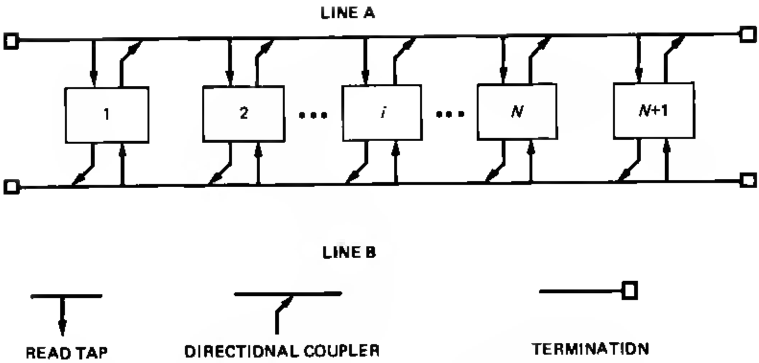

Fig. 1—Physical configuration of a Fasnet link.

The basic link as shown in Fig. 1 consists of two lines. One line carries traffic in one direction and the other line carries traffic in the reverse direction. For line A (which carries traffic from station 1 towards station N), station 1 is called the *head* station and station $N + 1$ is called the *end* station. For line B, the roles are reversed. Each station makes two connections to each line. A read tap precedes a passive directional coupler used for writing. The signal read from the read tap will be unaffected by the signal being written simultaneously on the line via the directional coupler. Except for specific fields of the header, the protocol ensures that only one station at a time writes on the line. Thus, once a message is written on a line, it is not removed or changed by any station.

The access control is similar for lines A and B. For line A, the head station (station 1) will initiate a *cycle*, which operates in the following way. One time each cycle, each station with packets destined toward the end station is allowed to access the line for a single time interval, during which at most p_{max} packets can be sent. A station knows when to place its packets on the line by reading a particular bit (called the busy bit) in the access control field. This bit is added to the message packet by the network layer of the protocol. Thus, in each cycle, station 1 has the first opportunity to send packets, station 2 has the second opportunity, and station N has the Nth and last* opportunity. Each station has exactly one opportunity per cycle to send packets. When station $N + 1$ receives a packet in which the busy bit indicates that the packet has not been used, it sends a message to station 1 (using line B) to start a new cycle. There may be synchronization delays at each end of the transmission of this message. The operation on line B is identical to the operation of line A, with station $N + 1$ as the head station and all flows reversed accordingly.

* We assume that station $N + 1$ will not send messages to itself.

III. THE MODEL

Fasnet behaves as an implicit token-passing protocol because control passes from station to station as if a token were sent from station 1 to station 2, $\cdots$, to station $N + 1$ to station 1, and so forth. This suggests that a queueing model of queues served in cyclic order would be appropriate. In this type of model, there is a single server that visits $N + 1$ different queues in the cyclic order described above. For Fasnet, the server is conceptual: it is the opportunity to place packets on the line. The service time is the length of time to write a packet. The queues correspond to the buffers at each station.

Several papers have been written about queues served in cyclic order. In most of these papers, it is assumed that there is *exhaustive service* at each station. This means that the server processes all customers waiting at the station at the epoch that the server reaches the station. The most general model of exhaustive service is in Eisenberg,[2] where each queue is of the M/G/1 type and the times to travel between adjacent stations may depend on the pair of stations involved. The solution of this model is in terms of transforms that are not given in closed form; however, the equations can be solved numerically. A special case of the model in Ref. 2 is treated in Konheim and Meister.[3] Here, all service times are the constant Δ, and all travel times between adjacent stations have the same distribution, which is concentrated on Δ, 2Δ, $\cdots$. Konheim and Meister are able to obtain closed-form solutions for steady-state performance measures in this case. Their results will be used in our analysis. The only paper where service is not exhaustive is Ref. 4 by Eisenberg. That paper contains two M/M/1 type queues, and the server can process at most one customer during a visit to a server. The solution to this model requires extensive calculations, and the restriction to two stations is unrealistic for Fasnet.

We have chosen to seek approximate solutions where each station is of the M/D/1 type and service is either exhaustive or one-at-a-time (as in Ref. 4). These correspond to $p_{max} = \infty$ and $p_{max} = 1$, respectively. When the system is not heavily loaded, each station will have a small load so $p_{max} = \infty$ should not behave much differently from $p_{max} = 1$. This behavior is exhibited by our approximate solution and by simulations.

We will analyze our models by embedding them in a server-vacation model. In an M/G/1/FIFO queue, assume that at the end of each busy period the server takes a vacation. The vacations are iid random variables generically denoted by T. The expected delay of a customer, in the steady state, is given by

$$E(D^*) = E(D) + \frac{E(T^2)}{2E(T)}. \tag{1}$$

Here $E(D)$ is the Pollaczek-Khintchine formula for the expected delay in an M/G/1 queue: $E(D) = ab_2/2(1 - ab_1)$, where a is the arrival rate and b_1 and b_2 are the first and second moments of the service times. A derivation of eq. (1) can he found in Levy and Yechiali.[5] Our first step is to describe the vacations.

3.1 Assumptions and notation

Tbe notation used in this paper and some self-explanatory symbols are given helow.

N = numher of potential transmitting stations.
λ_i = packet arrival rate at station i.
Δ = constant service time/packet.
$\rho_i = \lambda_i\Delta$ = load due to station i.
$R = \sum_1^N \rho_i$ = load on the line.
$\Lambda = \sum_1^N \lambda_i$ = total arrival rate.
τ = one-way propagation delay.
$\gamma = 2\tau + \Delta$ = average overhead/cycle.

Lines A and B shown in Fig. 1 are the same except for direction, so it is sufficient to model only line A. Notice that station $N + 1$ does not send messages on line A and that station 1 does not send messages on line B. We assume that packets to be transmitted appear at station i according to a homogeneous Poisson process with rate λ_i, $i = 1, 2, \cdots, N$. The arrivals at station i and j are independent when $i \neq j$. We assume tbat all the packets contain the same numher of bits. The amount of time that a line spends taking a packet from a station is the time required to read the hits of the packet. Therefore, the service time of each packet is a constant, Δ, ssy, where Δ is the number of hits/packet divided hy the line speed in b/s. The time for a bit to travel hetween adjacent read taps is called the *walk time*; the average walk time between stations is denoted hy w.

Let τ he the time to send a hit from station 1 to station $N + 1$ (and from $N + 1$ to 1). Then 2τ is expended in each cycle to send a message indicating that the husy bit must be reset. An average of one-half of a packet processing time is lost in synchronization at each end station, so $\gamma = 2\tau + \Delta$ is the average overhead per cycle. Notice that τ is the time to walk from station 1 to station $N + 1$.

3.2 Analysis of the exhaustive service model

In this subsection, we assume that the packet arrival rates are the same at each station. We let λ denote the common arrival rate and ρ denote the common load on a station. The load on the line is $R = N\rho$. We also assume that stations $1, 2, \cdots, N + 1$ are evenly spaced along the line.

Let T represent the amount of time between a departure from

station 1 and the beginning of the next visit to station 1. Then T is the length of a vacation from station 1. By symmetry, T is the length of a vacation between successive visits to any station. Thus, wben a station gains access to the line, an average of $\lambda \mathrm{E}(T)$ packets are present. Witb exhaustive service, the expected time to clear a station is $\lambda \mathrm{E}(T)\Delta/(1-\rho)$. Since a vacation from station 1 consists of reading packets at stations 2 through N and overbead,

$$\mathrm{E}(T) = \gamma + (N-1)\,\frac{\lambda \mathrm{E}(T)\Delta}{1-\rho},$$

so

$$\mathrm{E}(T) = \frac{\gamma(1-\rho)}{1-R}, \qquad R < 1. \tag{2}$$

Equation (2) is a special case of eq. (54) in Ref. 2.

Let X denote the number of packets at station 1 at the end of a vacation. Then

$$\mathrm{E}(X) = \lambda \mathrm{E}(T), \tag{3a}$$

and

$$\mathrm{Var}(X) = \lambda^2 \mathrm{Var}(T). \tag{3b}$$

Theorem 4.7 in Ref. 3 asserts that

$$\mathrm{Var}(X) = \frac{\lambda\rho^2[1-(N+1)\rho+(2N-1)\rho^2]}{(1-R)^2}. \tag{4}$$

From eqs. (2), (3), and (4) we obtain

$$\frac{\mathrm{E}(T^2)}{\mathrm{E}(T)} = \frac{\rho(N-1)\Delta + \gamma(1-\rho)^2}{(1-\rho)(1-R)}. \tag{5}$$

Equation (5) is exact when the walk times between any pair of stations are concentrated on $\Delta, 2\Delta, \cdots$. In Fasnet, we expect that the walk times are much less than Δ because the packets travel between stations at the speed of light and the read times are controlled by the speed of the line. Thus, we should regard eq. (5) as an approximation.

From eqs. (1) and (5),

$$\begin{aligned}\mathrm{E}(D^*) &= \frac{\rho\Delta}{2(1-\rho)} + \frac{\rho(N-1)\Delta}{2(1-\rho)(1-R)} + \frac{\gamma(1-\rho)}{2(1-R)} \\ &= \frac{R\Delta}{2(1-R)} + \frac{\gamma(1-\rho)}{2(1-R)}, \qquad R < 1.\end{aligned} \tag{6}$$

By symmetry, eq. (6) gives the expected delay of a packet at any station. The expected number of packets in a buffer, $\mathrm{E}(Q)$, say, is obtained from eq. (6) and Little's theorem:

$$\mathrm{E}(Q) = \lambda \mathrm{E}(D^*) = \frac{R\rho}{2(1-R)} + \frac{\gamma\lambda(1-\rho)}{2(1-R)}. \tag{7}$$

Consequently, the expected number of customers in buffers 1 through N is

$$N\mathrm{E}(Q) = \frac{R^2}{2(1-R)} + \frac{\gamma\Lambda(1-\rho)}{2(1-R)}. \tag{8}$$

Equation (7) is compared to simulation experiments in Section IV.

An intuitive understanding of eq. (6) may be gained by considering what happens for light loads. When ρ is small, the vacations are almost of constant length because (mostly) no customers are served during a vacation. Then $\mathrm{E}(T^2)$ is about $[\mathrm{E}(T)]^2$. Now merge all the customers to obtain an M/D/1 queue with mean delay $R\Delta/[2(1-R)]$. Then eqs. (1) and (2) yield

$$\mathrm{E}(D^*) = \frac{R\Delta}{2(1-R)} + \frac{\gamma(1-\rho)}{2(1-R)},$$

which is eq. (6). It is surprising that this heuristic light traffic argument produces the exact (modulo our other approximations) result for any $R < 1$.

3.3 Analysis of the one-at-a-time service model

In this subsection we will obtain an approximate solution to a model where $p_{\max} = 1$. The approximation is based on an idea used in Lehoczky, Sha, and Jensen[6] for a similar model. We do not assume that the arrival rates are the same at each station (as we did in Section 3.2).

A central notion in the approximation is the *completion time* of a station. The completion time of a station is the duration of the interval that starts when that station begins processing a packet, and ends at the first epoch that another packet may begin (does begin, provided it is present) its processing at the station. The purpose of the approximate solution is to estimate the mean and variance of the completion time, use these moments in the Pollaczek-Khintchine formula, and apply eq. (1) to an appropriate server-vacation model.

Let V_i denote a generic completion time at station i, in the steady state. Then

V_i = cycle overhead + Δ(1 + number of other stations sending a packet in this cycle).

Let p_i be the asymptotic proportion of time that packets are present at station i. For T very large, the number of packets served at station i by time T is

$$\frac{p_i T}{\mathrm{E}(V_i)} + \mathrm{o}(T),$$

where $\mathrm{o}(T)$ is some function such tbat $\mathrm{o}(T)/T \to 0$ as $T \to \infty$. The arrival rate at station i is λ_i. Equating the arrival and departure rates yields

$$p_i = \lambda_i \mathrm{E}(V_i). \tag{9}$$

Now we make our first approximation.

Approximation 1: In each cycle, the probability that station i transmits a packet is $b_i \triangleq \lambda_i \mathrm{E}(V_i)$.

The effect of approximation 1 is to use the long-run proportion p_i as a probability for each cycle. Thus, the expected number of other stations sending a packet during a completion time of station i is $\sum_{j \neq i} b_j$, so

$$\mathrm{E}(V_i) = \Delta \left(1 + \sum_{j \neq i} b_j\right) + \gamma. \tag{10}$$

Hence,

$$b_i \triangleq \lambda_i \mathrm{E}(V_i) = \rho_i \left(1 + \sum_{j \neq i} b_j\right) + \lambda_i \gamma, \qquad i = 1, 2, \cdots, N. \tag{11}$$

The solution of eq. (11) is

$$b_i = \frac{\rho_i}{1 + \rho_i} \frac{1 + \gamma/\Delta}{1 - \alpha}, \qquad \alpha = \sum_1^N \frac{\rho_i}{1 + \rho_i}, \qquad i = 1, 2, \cdots, N, \tag{12}$$

which can be verified by substitution into eq. (11). When $\rho_i \equiv \rho$ and $\gamma = 0$, eq. (12) becomes

$$b_i = \frac{\rho}{1 - (N - 1)\rho} = \frac{R/N}{1 - \dfrac{N - 1}{N} R} \qquad \text{for all } i,$$

which is the approximation given in Lehoczky et al.[6]

Since b_i must be no larger than one (because it is a probability), eq. (12) constrains the feasible values of $\{\rho_i\}$. When all the arrival rates have the common value λ, $\alpha = N\rho/(1 + \rho) = R/(1 + \rho)$, so

$$b_i < 1 \Leftrightarrow N > \frac{R\gamma/\Delta}{1 - R}. \tag{13}$$

Equation (13) shows that for a given total load, R, stability is achieved only when the load is shared by a sufficiently large number of stations. For Fasnet, $\gamma/\Delta = 3$ is a typical value. Then eq. (13) asserts that for $R = 0.9$, $N > 27$ is required for stability; for $R = 0.8$,

$N > 12$ is required for stability; and for $R = 0.5$, $N \geq 8$ is required for stability.

Here is why N cannot be too small. When N is small, the overhead per cycle (γ) will be spread over a few customers, which has the effect of decreasing the capacity of the line. To see this more precisely, let c be the expected number of packets processed in a cycle. Then use eq. (11) to obtain

$$c = \sum_1^N b_i = \frac{1 + \gamma/\Delta}{1 - \alpha} \sum_1^N \frac{\rho_i}{1 + \rho_i} = \frac{(1 + \gamma/\Delta)\alpha}{1 - \alpha}. \tag{14}$$

When all the arrival rates have the common value λ, eq. (14) yields

$$c = \frac{R(1 + \gamma/\Delta)}{1 + \rho - R}, \tag{15}$$

where $\rho = \lambda\Delta$. From eq. (15) we compute the average amount of overhead expended per packet per cycle* γ/c:

$$\frac{\gamma}{c} = \frac{1 - R + R/N}{R(1 + \gamma/\Delta)}\gamma = \frac{\gamma}{1 + \gamma/\Delta}\left(\frac{1 - R}{R} + \frac{1}{N}\right). \tag{16}$$

Eq. (16) shows that the average overhead per packet is a decreasing function of N, so if N were small, the overhead per packet might cause the line to be overloaded.

It is interesting to note that this consideration does not arise in Section 3.2. When $p_{max} < \infty$, from time to time a station will stop transmitting packets because its quota for the cycle has been filled. This is the effect that is shown in eq. (16). We conjecture that when all other parameters are fixed, γ/c is a decreasing function of p_{max}.

Now we turn to an approximation for the mean delay. This will be done by proposing a suitable server-vacation model and applying eq. (1). The service-time moments in the Pollaczek-Khintchine formula correspond to completion-time moments here. When a packet arrives at station i, and no other packets are waiting to be transmitted at station j, that packet cannot be transmitted until station i gains access to the line. The length of time that station i does not bave access to the line is the length of time to process the other stations in a cycle plus the overhead time, which is the completion time less one service time. Thus, $T_i = V_i - \Delta$, so

$$E(T_i) = E(V_i) - \Delta, \tag{17}$$

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* The fact that γ/c is the right quantity to compute may not be obvious. A rigorous proof could use ergodic theory for regenerative processes (see, e.g., Section 6-4 of Heyman and Sobel,[7] particularly Theorem 6-8).

and

$$\mathrm{Var}(T_i) = \mathrm{Var}(V_i). \tag{18}$$

From eqs. (10), (12), and (17) we can obtain $E(T_i)$. From eqs. (1) and (18) we see that only $\mathrm{Var}(V_i)$ remains to be obtained. To get it, we make our second approximation.

Approximation 2: In each cycle, the event that station i transmits a packet is independent of the event that station j transmits a packet for every $j \neq i$.

The effect of approximation 2 is that the variance of the number of packets served at station i is $b_i(1 - b_i)$ and the variance of the number of packets served in a cycle is $\sum_{i=1}^{N} b_i(1 - b_i)$. This yields the approximation

$$\mathrm{Var}(V_i) = \Delta^2 \sum_{j \neq i} b_j(1 - b_j). \tag{19}$$

Using eqs. (10), (12), (17), (18), and (19) in eq. (1) produces our approximation for the expected delay at station i, $i = 1, 2, \cdots, N$. The resulting formula does not appear to provide any insight and is omitted.

As a partial check on the efficacy of our approximation for the mean delay, we consider the limiting case of no overhead and identical stations. In this situation, the total content of the output buffers at stations 1, 2, $\cdots$ N fluctuates as the queue length in an M/D/1 queue with arrival rate λ and service time Δ. From tbe Pollaczek-Khintchine formula, the expected queue length in tbe steady state, $\mathrm{E}(Q_0)$, say, is

$$\mathrm{E}(Q_0) = \frac{R^2}{2(1 - R)}, \qquad R < 1.$$

Our approximations produce (after some algebra)

$$\mathrm{E}(\hat{Q}_0) = \frac{R^2}{2(1 - R + \rho)} \, \frac{\rho[R + (1 - R)(R - \rho)] + 1 - R}{1 - R}, \qquad R < 1.$$

Now let $N \to \infty$ and $\rho \downarrow 0$ with $R = N\rho$ beld fixed. This represents a system with many lightly loaded stations. Then

$$\mathrm{E}(\hat{Q}_0) \to \frac{R}{2(1 - R)} \qquad \text{as} \quad \rho \downarrow 0.$$

In this limiting case, $\mathrm{E}(\hat{Q}_0)$ overestimates $\mathrm{E}(Q_0)$ by $R/2$ and, the relative error is $(1 - R)/R$. Thus, the absolute error increases with R and is less than one-half, and the relative error decreases as the absolute error increases.

IV. COMPARISONS WITH SIMULATIONS

A computer simulation of Fasnet has been constructed. There are $N = 50$ sending stations equally spaced along the line. The propagation time (τ) equals the packet processing time at each station (Δ). We have chosen to consider 1000-bit packets and a line speed of 100 Mb/s, which is representative of the operating region. Then $\Delta = 10$ μs, and $\lambda = (R/50) \times 10^5$ packets/s.

The measure of performance is the average queue length at the stations. Specifically, the simulation estimates the steady-state distribution of the queue length at station i and then computes the mean, $E(Q_i)$, say. The average queue length is $\sum_1^{50} E(Q_i)/50 \triangleq E(Q)$. The corresponding quantity from our formulas is called $E(\hat{Q})$. From the queueing formula $E(Q) = \lambda E(D)$ we use $E(\hat{Q})$ to estimate the average delay of a packet, $E(\hat{D})$.

Table I shows the results.

The analytic approximation adequately replicates the simulation results. Table I and Fig. 2 demonstrate that for R as large as 0.8, $p_{\max} = 1$ and $p_{\max} = \infty$ produce nearly the same average queue size. This means that the efficiency (in terms of not incurring too much overhead) of $p_{\max} = \infty$ and the protection against a few stations dominating the line of $p_{\max} = 1$ can be simultaneously obtained by setting $1 < p_{\max} < \infty$. Table I shows that $p_{\max} = 3$ is almost as efficient as $p_{\max} = \infty$.

The approximation for the mean delay of a packet is less than 1/2 ms even when $R = 0.9$ and $p_{\max} = 1$.

V. THE EFFECTS OF BURSTY TRAFFIC

In this section we return to the exhaustive service model and replace the assumption that packets arrive according to a Poisson process with the assumption that packets arrive according to a compound Poisson process. Fuchs and Jackson give statistical analyses of arrival times for terminal-to-computer calls.[8] Two of their conclusions are as follows:

1. The exponential distribution is a reasonably good approximation of the times between bursts.

Table I—Comparison of simulations and analytic approximations

$p_{\max}$		$E(Q)$ Simulation	$E(\hat{Q})$ Analysis	$E(\hat{D})$
$R = 0.8$	1	0.216	0.209	130 μs
	3	0.184	—	—
	∞	0.184	0.150	94 μs
$R = 0.9$	1	0.684	0.782	430 μs
	3	0.440	—	
	∞	0.398	0.346	192 μs

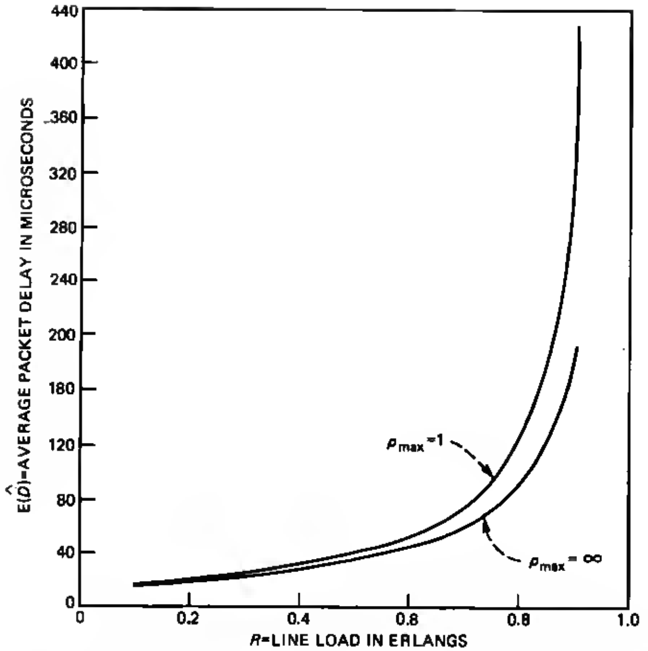

Fig. 2—Expected delay vs R for 1-kb packets, 50 stations, and 100-Mb/s line speed.

2. The size of a hurst (measured in various ways) has a geometric distribution.

Recent analyses by Morgan of host-to-host file traffic indicate that the assumption of Poisson arrivals may not be justified.[8]

The purpose of this section is to find out how sensitive the average delay is to the assumption of Poisson arrivals. We will see that in the exhaustive service model, the average delay can be significantly greater with bursty arrivals than with Poisson arrivals with the same rate.

Specifically, we assume that the bursts arrive according to a Poisson process with rate λ_b and the hurst sizes $B_1, B_2, \cdots$ are iid with

$$P\{B_1 = i\} = (1 - \xi)\xi^{i-1}, \qquad i = 1, 2, \cdots.$$

This arrival process can be interpreted as one where messages arrive according to a Poisson process with rate λ_b, and the jth message consists of a random number of packets with a geometric distrihution. One reason for choosing a geometric distribution is that it equates the average delay of a packet and the average delay of a message (see Halfin[10]). (The delay of a message is the delay of its last packet.) Another reason is that it uses only one parameter, and so we can

specify the mean (Mt, say) and variance (Vt, say) of the number of arrivals in an interval of length t and then solve for λ_b and ξ. Doing so yields

$$\lambda_b = \frac{2M^2}{M+V} \quad \text{and} \quad \xi = \frac{V-M}{V+M}.$$

Letting $z = V/M \geq 1$ yields

$$\lambda_b = \frac{2M}{1+z} \quad \text{and} \quad \frac{z-1}{z+1}. \tag{20}$$

Equation (20) relates the parameters we might obtain from measurements, M and z, to the parameters of the model, λ_b and ξ.

We will now obtain the delay of an arbitrary packet in the steady state. The analysis is similar to the analysis in subsection 3.2; as before we assume that the stations have statistically identical arrival processes.

The analog of the Pollaczek-Khintchine formula for compound Poisson arrivals is given in Burke.[11] In our notation, the formula is

$$\mathrm{E}(D) = \frac{\lambda_b \Delta^2 (1+\xi)}{2(1-\rho)} + \frac{\xi \Delta}{1-\xi}, \tag{21}$$

where $\rho = \lambda_b \Delta/(1-\xi) = M\Delta$.

To obtain the mean and variance of the vacation times, let X denote the number of packets at station 1 at the end of a vacation. From theorem 4.7 in Ref. 3, for $R < 1$

$$\mathrm{E}(X) = \frac{\gamma M(1-\rho)}{1-R}, \tag{22a}$$

and

$$\mathrm{Var}(X) = \frac{XV}{(1-R)^2}\,[1-(N+1)\rho + (2N-1)\rho^2]. \tag{22b}$$

Since X is the number of packets that arrive in an interval of length T,

$$\mathrm{E}(X) = M\mathrm{E}(T), \tag{23a}$$

and

$$\begin{aligned} \mathrm{Var}(X) &= \mathrm{E}[\mathrm{Var}(X \mid T)] + \mathrm{Var}[\mathrm{E}(X \mid T)] \\ &= \mathrm{E}[VT] + \mathrm{Var}[MT] \\ &= V\mathrm{E}(T) + M^2\mathrm{Var}(T). \end{aligned} \tag{23b}$$

From eqs. (22) and (23) we obtain

$$\frac{E(T^2)}{E(T)} = \frac{\mathrm{Var}(X)}{ME(X)} - \frac{V}{M^2} + E(T)$$

$$= \frac{(N-1)V\Delta^2 + \gamma(1-\rho)^2}{(1-\rho)(1-R)}. \tag{24}$$

Substituting eqs. (21) and (24) into eq. (1) yields

$$E(D^*) = \frac{\lambda_b \Delta^2(1+\xi)}{2(1-\rho)} + \frac{\xi\Delta}{1-\xi} + \frac{(N-1)V\Delta^2 + \gamma(1-\rho)^2}{2(1-\rho)(1-R)}.$$

Using eq. (20) yields

$$E(D^*) = \frac{R\Delta z}{2(1-R)} + \frac{\gamma(1-\rho)}{2(1-R)} + \frac{(z-1)\Delta}{2} - \frac{\rho\Delta z(z-1)}{2(1-\rho)(1+z)}. \tag{25}$$

To compare eqs. (6) and (25), let a subscript z denote batcb arrivals with variance to mean ratio z. Then

$$E(D_z^*) - E(D^*) = \frac{\Delta(z-1)}{2(1-R)} - \frac{\rho\Delta z(z-1)}{2(1-\rho)(1+z)}.$$

When N is large, so that ρ is much smaller tban R, the first term dominates, especially in heavy traffic.

Table II sbows the values of $E(D_z^*)$. The case $E(D_1^*)$ represents Poisson arrivals. The data are the same as in Section IV: $\Delta = \tau = 10$ μs, and $N = 50$.

Table II shows that bursty traffic can have much larger expected delays than Poisson traffic with the same arrival rate. A crude approximation of the increase is $E(D_z^*) = E(D_1^*)\sqrt{z-1}$ for $2 \leq z \leq 10$. Even when $z = 10$ and $R = 0.9$, the mean delay is less tban 1 ms.

VI. CONCLUSIONS

We have three conclusions. The first is that the approximations presented in Section III are sufficiently accurate for data transport performance studies of Fasnet. The second is that $p_{max} = 3$ appears to be a good choice if the offered traffic is reasonably smooth (Poisson) and approximately equal to all stations. The third is that Fasnet should be able to provide 1-ms average-delay performance for bursty traffic.

Table II—$E(D_z^*)$ in μs for several values of z

	$E(D_1^*)$	$E(D_2^*)$	$E(D_5^*)$	$E(D_{10}^*)$
$R = 0.8$	94	119	193	318
$R = 0.9$	192	242	392	642

VII. ACKNOWLEDGMENTS

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